OCR
Oxford Cambridge and RSA

Wednesday 8 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4757/01 Further Applications of Advanced Mathematics (FP3)

QUESTION PAPER

6385952923

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4757/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **24** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Option 1: Vectors

1 Positions in space around an aerodrome are modelled by a coordinate system with a point on the runway as the origin, O. The x-axis is east, the y-axis is north and the z-axis is vertically upwards. Units of distance are kilometres. Units of time are hours.

At time $t = 0$, an aeroplane, P, is at $(3, 4, 8)$ and is travelling in a direction $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ at a constant speed of $900\,\mathrm{km\,h^{-1}}$.

(i) Find the least distance of the path of P from the point O. **[4]**

At time $t = 0$, a second aeroplane, Q, is at (80, 40, 10). It is travelling in a straight line towards the point O. Its speed is constant at $270\,\mathrm{km\,h^{-1}}$.

(ii) Show that the shortest distance between the paths of the two aeroplanes is 2.24 km correct to three significant figures. **[6]**

(iii) By finding the points on the paths where the shortest distance occurs and the times at which the aeroplanes are at these points, show that in fact the aeroplanes are never this close. **[7]**

(iv) A third aeroplane, R, is at position (29, 19, 5.5) at time $t = 0$ and is travelling at $285\,\mathrm{km\,h^{-1}}$ in a direction $\begin{pmatrix} 18 \\ 6 \\ 1 \end{pmatrix}$. Given that Q is in the process of landing and cannot change course, show that R needs to be instructed to alter course or change speed. **[7]**

Option 2: Multi-variable calculus

2 A surface, S, has equation $z = 3x^2 + 6xy + y^3$.

(i) Find the equation of the section where $y = 1$ in the form $z = \mathrm{f}(x)$. Sketch this section.

Find in three-dimensional vector form the equation of the line of symmetry of this section. **[5]**

(ii) Show that there are two stationary points on S, at O (0, 0, 0) and at P (−2, 2, −4). **[4]**

(iii) Given that the point $(-2 + h, 2 + k, \lambda)$ lies on the surface, show that

$$\lambda = -4 + 3(h + k)^2 + k^2\,(k + 3).$$

By considering small values of h and k, deduce that there is a local minimum at P. **[5]**

(iv) By considering small values of x and y, show that the stationary point at O is neither a maximum nor a minimum. **[3]**

(v) Given that $18x + 18y - z = d$ is a tangent plane to S, find the two possible values of d. **[7]**

Option 3: Differential geometry

3 Fig. 3 shows the curve with parametric equations $x = t - 3t^3,\ y = 1 + 3t^2$.

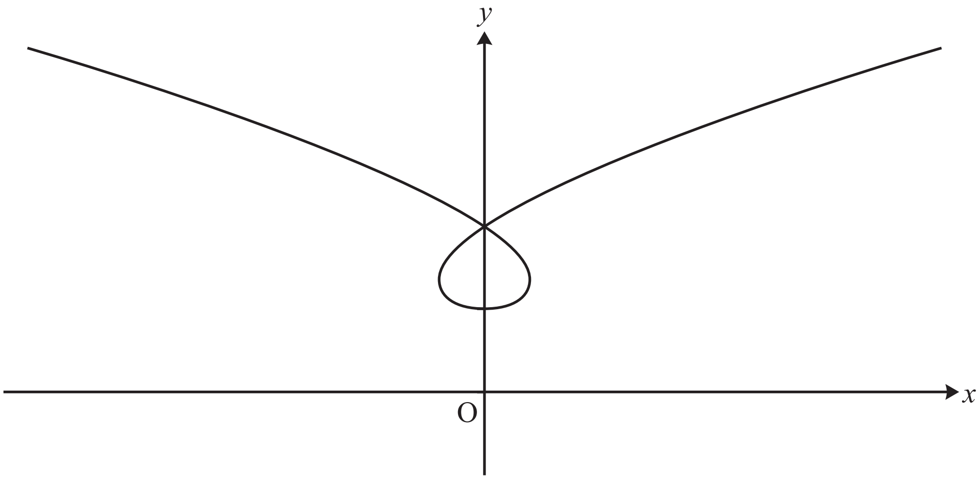

Fig. 3

(i) Show that the values of t where the curve cuts the y-axis are $t = 0, \pm\frac{1}{\sqrt{3}}$. Write down the corresponding values of y. **[2]**

(ii) Find the radius and centre of curvature when $t = \frac{1}{\sqrt{3}}$. **[11]**

The arc of the curve given by $0 \leqslant t \leqslant \frac{1}{\sqrt{3}}$ is denoted by C.

(iii) Find the length of C. **[5]**

(iv) Show that the area of the curved surface generated when C is rotated about the y-axis through 2π radians is $\frac{\pi}{3}$. **[6]**

Option 4: Groups

4 **(a)** The elements of the set $P = \{1, 3, 9, 11\}$ are combined under the binary operation, *, defined as multiplication modulo 16.

(i) Demonstrate associativity for the elements 3, 9, 11 in that order.

Assuming associativity holds in general, show that P forms a group under the binary operation *. **[6]**

(ii) Write down the order of each element. **[2]**

(iii) Write down all subgroups of P. **[1]**

(iv) Show that the group in part **(i)** is cyclic. **[1]**

(b) Now consider a group of order 4 containing the identity element e and the two distinct elements, a and b, where $a^2 = b^2 = e$. Construct the composition table. Show that the group is non-cyclic. **[4]**

(c) Now consider the four matrices **I**, **X**, **Y** and **Z** where

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{X} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \mathbf{Y} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{Z} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The group G consists of the set $\{\mathbf{I}, \mathbf{X}, \mathbf{Y}, \mathbf{Z}\}$ with binary operation matrix multiplication. Determine which of the groups in parts **(a)** and **(b)** is isomorphic to G, and specify the isomorphism. **[6]**

(d) The distinct elements $\{p, q, r, s\}$ are combined under the binary operation $\circ$. You are given that $p \circ q = r$ and $q \circ p = s$.

By reference to the group axioms, prove that $\{p, q, r, s\}$ is not a group under $\circ$. **[4]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 Each day that Adam is at work he carries out one of three tasks A, B or C. Each task takes a whole day. Adam chooses the task to carry out on each day according to the following set of three rules.

1. If, on any given day, he has worked on task A then the next day he will choose task A with probability 0.75, and tasks B and C with equal probability.
2. If, on any given day, he has worked on task B then the next day he will choose task B or task C with equal probability but will never choose task A.
3. If, on any given day, he has worked on task C then the next day he will choose task A with probability p and tasks B and C with equal probability.

(i) Write down the transition matrix. **[3]**

(ii) Over a long period Adam carries out the tasks A, B and C with equal frequency. Find the value of p. **[4]**

(iii) On day 1 Adam chooses task A. Find the probability that he also chooses task A on day 5. **[3]**

Adam decides to change rule 3 as follows.

If, on any given day, he has worked on task C then the next day he will choose tasks A, B, C with probabilities 0.4, 0.3, 0.3 respectively.

(iv) On day 1 Adam chooses task A. Find the probability that he chooses the same task on day 7 as he did on day 4. **[5]**

(v) On a particular day, Adam chooses task A. Find the expected number of consecutive further days on which he will choose A. **[3]**

Adam changes all three rules again as follows.

- If he works on A one day then on the next day he chooses C.
- If he works on B one day then on the next day he chooses A or C each with probability 0.5.
- If he works on C one day then on the next day he chooses A or B each with probability 0.5.

(vi) Find the long term probabilities for each task. **[6]**

END OF QUESTION PAPER

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Centre number						Candidate number				

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Turn over

1 (i)

1 (ii)	

1 (iii)	

1 (iv)

2 (i)	

2 (ii)	

2 (iii)	
2 (iv)	

2 (v)

3 (i)

3 (ii)

continued on next page

3 (ii)	**Continued**

3 (iii)

3 (iv)	

4 (a)(i)	
4 (a)(ii)	
4 (a)(iii)	
4 (a)(iv)	

4 (b)	

4 (c)

4 (d)

5 (i)	

5 (ii)	

5 (iii)

5 (iv)

5 (v)	

5 (vi)	

Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4757**: Further Applications of Advanced Mathematics

Advanced GCE

Mark Scheme for June 2016

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

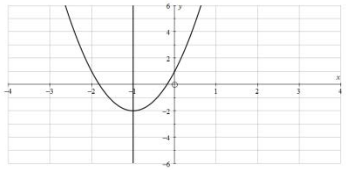
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		$\begin{pmatrix}3\\4\\8\end{pmatrix}\times\begin{pmatrix}2\\1\\0\end{pmatrix}=\begin{pmatrix}-8\\16\\-5\end{pmatrix}$	M1 A1	Appropriate vector product Correctly evaluated	
			Distance is $\dfrac{\sqrt{8^2+16^2+5^2}}{\sqrt{2^2+1^2+0^2}}=\dfrac{\sqrt{345}}{\sqrt{5}}$ $=\sqrt{69}\quad(\approx 8.31)\ \text{(km)}$	A1 A1	Dividing by $\sqrt{2^2+1^2+0^2}$	*Sign error in vector product can earn M1A0A1A1*
		OR	$\begin{pmatrix}3+2\lambda\\4+\lambda\\8\end{pmatrix}$ is perpendicular to $\begin{pmatrix}2\\1\\0\end{pmatrix}$		M1	
			$2(3+2\lambda)+(4+\lambda)=0$		A1	
			$\lambda=-2$; shortest vector is $\begin{pmatrix}-1\\2\\8\end{pmatrix}$		A1	
			Distance is $\sqrt{1^2+2^2+8^2}=\sqrt{69}$		A1	
				[4]		
	(ii)		$\begin{pmatrix}2\\1\\0\end{pmatrix}\times\begin{pmatrix}-8\\-4\\-1\end{pmatrix}=\begin{pmatrix}-1\\2\\0\end{pmatrix}$	M1 A1	Vector product of directions Correctly evaluated	
			$\begin{pmatrix}77\\36\\2\end{pmatrix}.\begin{pmatrix}-1\\2\\0\end{pmatrix}=-5\qquad\left[\text{ or }\begin{pmatrix}3\\4\\8\end{pmatrix}.\begin{pmatrix}-1\\2\\0\end{pmatrix}\right]$	M1 A1	Appropriate scalar product For (−) 5	*Dependent on previous M1*
			Distance is $\dfrac{5}{\sqrt{1^2+2^2+0^2}}$	M1	Dividing by $\sqrt{1^2+2^2+0^2}$	*Dependent on M1M1*
			$=\dfrac{5}{\sqrt{5}}=\sqrt{5}=2.236\ldots=2.24\ \text{(km)}$ (correct to 3 sf)	E1		
				[6]		

Question			Answer	Marks	Guidance	
	(iii)		$\overrightarrow{AB} = \begin{pmatrix} 80-8\mu \\ 40-4\mu \\ 10-\mu \end{pmatrix} - \begin{pmatrix} 3+2\lambda \\ 4+\lambda \\ 8 \end{pmatrix} \quad \left[= \begin{pmatrix} 77-2\lambda-8\mu \\ 36-\lambda-4\mu \\ 2-\mu \end{pmatrix} \right]$	B1		
			$\overrightarrow{AB}.\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = 0$ and $\overrightarrow{AB}.\begin{pmatrix} -8 \\ -4 \\ -1 \end{pmatrix} = 0$	M1		
			$2(77-2\lambda-8\mu)+(36-\lambda-4\mu)=0$ $-8(77-2\lambda-8\mu)-4(36-\lambda-4\mu)-(2-\mu)=0$	A1		
			$5\lambda+20\mu=190$ $20\lambda+81\mu=762$ and hence $\lambda=30,\ \mu=2$	A1		
		OR	$\overrightarrow{AB}$ is parallel to $\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$		M1	
			$36-\lambda-4\mu=-2(77-2\lambda-8\mu)$ $2-\mu=0$		A1	
			$5\lambda+20\mu=190$ $2-\mu=0$ and hence $\lambda=30,\ \mu=2$		A1	
		OR	$\overrightarrow{AB} = (\pm)\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$		M1	
			$77-2\lambda-8\mu=1$ $36-\lambda-4\mu=-2$ $2-\mu=0$		A1	
			$\lambda=30,\ \mu=2$		A1	

Question			Answer	Marks	Guidance	
			Closest points are A(63, 34, 8) and B(64, 32, 8)	A1		
			P is at A at time $t_1 = \dfrac{\sqrt{5}\lambda}{900} \left[\text{or } \dfrac{\sqrt{60^2+30^2+0^2}}{900} = \dfrac{\sqrt{4500}}{900} \right] = \dfrac{\sqrt{5}}{30}$	M1	Method for finding time	
			Q is at B at time $t_2 = \dfrac{9\mu}{270} \left[\text{or } \dfrac{\sqrt{16^2+8^2+2^2}}{270} = \dfrac{18}{270} \right] = \dfrac{2}{30}$ These times are different, so the planes are never this close	E1	Both times correct, and conclusion	
				[7]		

Question			Answer	Marks	Guidance	
	(iv)		$\mathbf{q}=\begin{pmatrix}80\\40\\10\end{pmatrix}+\frac{270t}{9}\begin{pmatrix}-8\\-4\\-1\end{pmatrix}$ $\left[=\begin{pmatrix}80-240t\\40-120t\\10-30t\end{pmatrix}\right]$	M1 A1	Speed and unit direction vectors	
			$\mathbf{r}=\begin{pmatrix}29\\19\\5.5\end{pmatrix}+\frac{285t}{19}\begin{pmatrix}18\\6\\1\end{pmatrix}$ $\left[=\begin{pmatrix}29+270t\\19+90t\\5.5+15t\end{pmatrix}\right]$	A1		
			Q, R will collide if $80-240t=29+270t$ $40-120t=19+90t$ $10-30t=5.5+15t$	M1	One equation sufficient for M1	
			All three equations have solution $t=0.1$ Planes would collide; so R must alter course or speed	A1 A1 E1	For $t=0.1$ obtained Shown to satisfy all three Correctly shown	Point of collision is (56, 28, 7)
		OR	Paths intersect if $80-8\mu=29+18\nu$ $40-4\mu=19+6\nu$ $10-\mu=5.5+\nu$		M1 OR A1 Two correct equations	$\Delta=\begin{pmatrix}-51\\-21\\-4.5\end{pmatrix}.\left[\begin{pmatrix}-8\\-4\\-1\end{pmatrix}\times\begin{pmatrix}18\\6\\1\end{pmatrix}\right]$
			$\mu=3,\ \nu=1.5$		A1	$\begin{pmatrix}-8\\-4\\-1\end{pmatrix}\times\begin{pmatrix}18\\6\\1\end{pmatrix}=\begin{pmatrix}2\\-10\\24\end{pmatrix}$
			All equations are satisfied, so paths intersect [at X (56, 28, 7)]		A1 *Must check third equation*	$\Delta=0$, so the paths intersect
			Q is at X at time $t=\frac{9\mu}{270}\left[\text{or }\frac{\sqrt{24^2+12^2+3^2}}{270}=\frac{27}{270}\right]=0.1$		M1 Method for finding time A1 For $t=0.1$	
			R is at X at time $t=\frac{19\nu}{285}\left[\text{or }\frac{\sqrt{27^2+9^2+1.5^2}}{285}=\frac{28.5}{285}\right]=0.1$			
			Planes would collide; so R must alter course or speed		E1 Correctly shown	*Dependent on all previous marks*
				[7]		

Question			Answer	Marks	Guidance	
2	(i)			B1	Correct shape	
				B1	Minimum in third quadrant and positive intercept on *z*-axis	
			$y=1 \Rightarrow z=3x^2+6x+1 \quad [=3(x+1)^2-2]$	B1		
			So line of symmetry is $\mathbf{r} = \begin{pmatrix}-1\\1\\-2\end{pmatrix} + \lambda\begin{pmatrix}0\\0\\1\end{pmatrix}$	B1B1	For $\begin{pmatrix}-1\\1\\.\end{pmatrix}$ and $\lambda\begin{pmatrix}0\\0\\1\end{pmatrix}$	
				[5]		
2	(ii)		We require $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = 0$	M1		
			$\frac{\partial z}{\partial x} = 6x+6y=0 \quad [\Rightarrow y=-x]$	A1	For either equation	
			$\frac{\partial z}{\partial y} = 6x+3y^2=0$ and hence $y^2=2y$	A1	Correct equation in *y* or *x*	Or $6x+3x^2=0$
			$y=0,2;\ x=0,-2;\ z=0,-4$ Stationary points are $(0,0,0)$ and $(-2,2,-4)$	E1	*Some working required for -4*	
				[4]		

Question			Answer	Marks	Guidance	
	(iii)		For $x=-2+h,\ y=2+k,\ z=\lambda$ $\lambda=3(-2+h)^2+6(-2+h)(2+k)+(2+k)^3$ $=12-12h+3h^2+6hk+12h-12k-24$ $+8+12k+6k^2+k^3$ $=-4+3h^2+6hk+6k^2+k^3$ $=-4+3(h+k)^2+k^3+3k^2$ $=-4+3(h+k)^2+k^2(k+3)$ Since $3(h+k)^2>0$ and $k^2(k+3)>0$ for small k $\lambda>-4$ for all small values of h and k so P is a minimum.	M1 A1 E1 M1 E1	Substitution *M0 for numerical work* Must mention small k or $k>-3$	
				[5]		
	(iv)		For small x and y, z can be positive or negative If $x=0$ and $y>0$, then $z>0$ If $x=0$ and $y<0$, then $z<0$ Hence O is neither a maximum nor a minimum	M1 A1 E1	(Numerical demonstration can earn M1A0E0) Correct argument which applies arbitrarily close to O Correctly shown	When $x=0,\ z=y^3$ which has a point of inflection
				[3]		
	(v)		We require $\frac{\partial z}{\partial x}=18,\ \frac{\partial z}{\partial y}=18$ $6x+6y=18,\ 6x+3y^2=18$ $2(3-y)+y^2=6$ Points are $(3, 0, 27)$ and $(1, 2, 23)$ $18x+18y-z=d$ So $d=27, 31$	M1 M1 A1 M1 A1 M1 A1	Allow -18 for M1 Obtaining equation for y or x or $2x+(3-x)^2=6$ Obtaining values of x, y, z	
				[7]		

Question			Answer	Marks	Guidance	
3	(i)		When $x=0,\ \ t=0,\pm\frac{1}{\sqrt{3}}$	E1		
			$y=1,\ \ y=2$	B1	For both	
				[2]		
	(ii)		$\dot{x}=1-9t^2,\ \dot{y}=6t \qquad \ddot{x}=-18t,\ \ddot{y}=6$	B1	All 4 soi	$\dot{x}=-2,\ \dot{y}=\frac{6}{\sqrt{3}} \qquad \ddot{x}=-\frac{18}{\sqrt{3}},\ \ddot{y}=6$
			$\rho=\frac{\left(\left(1-9t^2\right)^2+36t^2\right)^{3/2}}{6(1-9t^2)+108t^2}=\frac{\left(1+9t^2\right)^3}{6\left(1+9t^2\right)}=\frac{\left(1+9t^2\right)^2}{6}$	M1 A1	Use of formula for ρ or κ Unsimplified	$\rho=\frac{\left(4+12\right)^{3/2}}{-12+36}$
			When $t=\frac{1}{\sqrt{3}},\ \rho=\frac{16}{6}=\frac{8}{3}$	A1		
			$\tan\psi=\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6t}{1-9t^2}$	M1 A1		$\tan\psi=\frac{\mathrm{d}y}{\mathrm{d}x}=-\sqrt{3}$
			$\sin\psi=\frac{6t}{1+9t^2},\ \ \cos\psi=\frac{1-9t^2}{1+9t^2}$	M1 A1	or unit normal is $\begin{pmatrix}\frac{\sqrt{3}}{2}\\ \frac{1}{2}\end{pmatrix}$	$\sin\psi=\frac{\sqrt{3}}{2},\ \ \cos\psi=-\frac{1}{2}$
			Centre of curvature is at $\left(0-\frac{8}{3}\times\frac{\sqrt{3}}{2},\ \ 2-\frac{8}{3}\times\frac{1}{2}\right)$	M1		
			i.e. $\left(-\frac{4\sqrt{3}}{3},\ \frac{2}{3}\right)$	A1A1		
				[11]		
	(iii)		$\dot{x}^2+\dot{y}^2=\left(1-9t^2\right)^2+\left(6t\right)^2=\left(1+9t^2\right)^2$	M1A1	Soi	
			$s=\int_0^{1/\sqrt{3}}\left(1+9t^2\right)\mathrm{d}t$	M1	*Limits not required*	
			$=\left[t+3t^3\right]_0^{1/\sqrt{3}}$	A1	*Including limits*	
			$=\frac{2}{\sqrt{3}}=\frac{2}{3}\sqrt{3}$	A1		
				[5]		

Question			Answer	Marks	Guidance	
	(iv)		$S = 2\pi \int_0^{1/\sqrt{3}} x \, ds = 2\pi \int_0^{1/\sqrt{3}} \left(t - 3t^3\right)\left(1 + 9t^2\right) dt$	M1 M1 A1	Correct formula Integral in terms of t *Including limits*	
			$= 2\pi \int_0^{1/\sqrt{3}} \left(t + 6t^3 - 27t^5\right) dt = 2\pi \left[\frac{t^2}{2} + \frac{3}{2}t^4 - \frac{9}{2}t^6\right]_0^{1/\sqrt{3}}$	M1 A1	Expand and integrate *Including limits*	
			$= 2\pi\left(\frac{1}{6} + \frac{1}{6} - \frac{1}{6}\right) = \frac{\pi}{3}$	E1	*Intermediate step required*	
				[6]		

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th colspan="2">Guidance</th></tr>
<tr><td>4</td><td>(a)</td><td>(i)</td><td>3*(9*11) = 3*3 = 9
(3*9)*11 = 11*11 = 9

Construction of group table (or otherwise):

It shows closure,
the identity is 1
each element has an inverse
$3^{-1} = 11,\ 9^{-1} = 9, 11^{-1} = 3, 1^{-1} = 1$</td><td>B1
B1

B1

B1
B1

B1</td><td></td><td>Group table is:
$$\begin{array}{c|cccc} & 1 & 3 & 9 & 11 \\ \hline 1 & 1 & 3 & 9 & 11 \\ 3 & 3 & 9 & 11 & 1 \\ 9 & 9 & 11 & 1 & 3 \\ 11 & 11 & 1 & 3 & 9 \end{array}$$</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[6]</td><td></td><td></td></tr>
<tr><td></td><td></td><td>(ii)</td><td><table><tr><td>Element</td><td>1</td><td>3</td><td>9</td><td>11</td></tr><tr><td>Order</td><td>1</td><td>4</td><td>2</td><td>4</td></tr></table></td><td>B2</td><td>−1 each error</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[2]</td><td></td><td></td></tr>
<tr><td></td><td></td><td>(iii)</td><td>{1} {1,9} {1,3,9,11}</td><td>B1</td><td>Condone omission of trivial subgroups</td><td>B0 if any extras</td></tr>
<tr><td></td><td></td><td></td><td></td><td>[1]</td><td></td><td></td></tr>
<tr><td></td><td></td><td>(iv)</td><td>e.g. $3^2 = 9,\ 3^3 = 11, 3^4 = 1$
3 generates the group and so it is cyclic</td><td>E1</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[1]</td><td></td><td></td></tr>
<tr><td></td><td>(b)</td><td></td><td>Composition table:
$$\begin{array}{c|cccc} & e & a & b & ab \\ \hline e & e & a & b & ab \\ a & a & e & ab & b \\ b & b & ab & e & a \\ ab & ab & b & a & e \end{array}$$
All elements are self-inverse, and so no element generates the group</td><td>B3

E1</td><td>−1 each error</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td>[4]</td><td></td><td></td></tr>
</table>

Question			Answer	Marks	Guidance	
	(c)		In group G all elements are self-inverse i.e. $X^2 = I$, $Y^2 = I$ and $Z^2 = I$ So this group is isomorphic to the group in (b) e.g. $\mathbf{I} \leftrightarrow e \quad \mathbf{X} \leftrightarrow a \quad \mathbf{Y} \leftrightarrow b \quad \mathbf{Z} \leftrightarrow ab$	M1 A1A1 A1 B1B1	Correctly shown	
				[6]		
	(d)		One of the elements needs to be the identity element. It is neither p nor q for otherwise $p \circ q = p$ (or q) It is neither r nor s, for otherwise $p \circ q = q \circ p = r$ (or s) So there is no identity element and so not a group	M1 A1 A1 E1		
				[4]		

Question			Answer	Marks	Guidance	
5	(i)		$\begin{pmatrix} 0.75 & 0 & p \\ 0.125 & 0.5 & \frac{1-p}{2} \\ 0.125 & 0.5 & \frac{1-p}{2} \end{pmatrix}$	B1 M1 A1	1st two columns Making 3rd column sum to 1	
				[3]		
	(ii)		$\begin{pmatrix} 0.75 & 0 & p \\ 0.125 & 0.5 & \frac{1-p}{2} \\ 0.125 & 0.5 & \frac{1-p}{2} \end{pmatrix}\begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix}$ $0.75 + p = 1$ $p = 0.25$	M1 A1 A1 A1	Equilibrium probs Equation *Correct equation implies* *M1A1A1* *Just answer: B4*	
				[4]		
	(iii)		$\text{P(A on day 5)} = \begin{pmatrix} 0.75 & 0 & 0.25 \\ 0.125 & 0.5 & 0.375 \\ 0.125 & 0.5 & 0.375 \end{pmatrix}^4 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $= \begin{pmatrix} 0.435059 & ..... & ..... \\ ..... & ..... & ..... \\ ..... & ..... & ..... \end{pmatrix}$ $= 0.435$	M1 A1 A1	For power 4 At least one right, soi *Just answer: B3*	
				[3]		

Question			Answer	Marks	Guidance	
	(iv)		$\mathbf{P}=\begin{pmatrix}0.75 & 0 & 0.4\\ 0.125 & 0.5 & 0.3\\ 0.125 & 0.5 & 0.3\end{pmatrix}$, $\mathbf{P}^3=\begin{pmatrix}0.536875 & 0.31 & 0.431\\ 0.231563 & 0.345 & 0.2845\\ 0.231563 & 0.345 & 0.2845\end{pmatrix}$	M1	For using P^3	
			$\mathbf{P}^3\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}0.536875\\0.231563\\0.231563\end{pmatrix}$	M1 A1	First column of P^3	
			$p=0.536875\times0.536875+0.345\times0.2315625+0.2845\times0.2315625$ $=0.434(003)...$	M1 A1		
				[5]		
	(v)		P(from A to A) = 0.75 so $\alpha=0.75$ Expected number is $\frac{\alpha}{1-\alpha}=\frac{0.75}{0.25}=3$	B1 M1 A1	Using $\frac{\alpha}{1-\alpha}$	
				[3]		
	(vi)		$\begin{pmatrix}0 & 0.5 & 0.5\\ 0 & 0 & 0.5\\ 1 & 0.5 & 0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}x\\y\\z\end{pmatrix}$	B1	New transition matrix	
			$0.5y+0.5z=x,\quad 0.5z=y,\quad x+0.5y=z$	M1 A1		
			$x+y+z=1$	M1		
			$x=\frac{1}{3},\ y=\frac{2}{9},\ z=\frac{4}{9}$	A2	−1 each error	
		OR	New transition matrix Considering a high power (at least 20) P(A) = 0.333, P(B) = 0.222, P(C) = 0.444		B1 M2 Give M1 for at least 10 A1A1A1	
				[6]		

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4757 Further Applications of Advanced Mathematics (FP3)

General Comments:

Most candidates for this paper were able to produce substantial attempts at all three of their chosen questions. Q.2 (on multi-variable calculus) was the most popular question, chosen by over 80% of the candidates. Q.1 (on vectors), Q.4 (on groups) and Q.5 (on Markov chains) were each chosen by about 60% of the candidates. The least popular question was Q.3 (on differential geometry), which was chosen by fewer than 40% of the candidates.

Comments on Individual Questions:

Q.1(i) This was very well answered, with most candidates using the standard formula involving the magnitude of a vector product.

Q.1(ii) Most candidates knew how to find the shortest distance between the two paths, almost always using a scalar triple product.

Q.1(iii) Candidates used a variety of methods to find the points where the shortest distance occurred. Some applied scalar products of the general chord with the directions of the two paths to obtain two simultaneous equations. Some put the general chord parallel to the common perpendicular, which had already been found in part (ii); this was particularly efficient in this case, as the z component was zero. Another approach, quite often used successfully, was to take the general point on one path and put its shortest distance from the other path equal to $\sqrt{5}$, applying the formula from part (i). However, only a minority of candidates succeeded in finding the two points. Many tried putting the length of the general chord equal to $\sqrt{5}$, but the resulting equation proved too difficult to solve. Several candidates found an expression for the distance between the two aeroplanes in terms of time; this is a valid approach for showing that the aeroplanes never came as close as 2.24 km, but it does not answer the question asked and could not earn more than 2 marks (out of 7).

Q.1(iv) The simplest approach was to find expressions for the position vectors of Q and R at time t and then show that these position vectors were equal when $t = 0.1$. A more common approach was to show that the paths of Q and R intersect, either by evaluating a scalar triple product or by equating the components of general points on the two lines. Very many of the latter group omitted to check that the three equations were consistent, and so had not actually shown that the lines did intersect, even though they had found the correct point of intersection. It was then necessary to show that both aeroplanes reached the point of intersection at the same time; but many candidates omitted this step.

Q.2(i) The section was usually drawn correctly. For the line of symmetry, many candidates gave an answer which was not recognisable as the three-dimensional vector equation of a line. A common error was to give the normal line to S, rather than the line of symmetry of the section.

Q.2(ii) Most candidates found the partial derivatives and the stationary points correctly. As the answers were given it was necessary to show sufficient working, and many candidates lost a mark by not showing the calculation of the z coordinate of P.

Q.2(iii) This involved substituting into the equation of the surface and rearranging, which was usually done correctly. Some candidates tried to use an approximate result for small changes using the partial derivatives, although an exact result was required here. Many candidates did not then show convincingly that $\lambda > -4$ for all small values of h and k; some omitted this part, and some resorted to substituting numerical values.

Q.2(iv) Most candidates tried to show that z can take both positive and negative values close to O, but few produced a convincing argument which applied arbitrarily close to O. Many candidates wrote '$3x^2$ is always positive, $6xy$ and y^3 can be positive or negative, therefore $3x^2 + 6xy + y^3$ can be positive or negative', which was not quite sufficient. The simplest way was to consider the section given by $x = 0$, which was $z = y^3$.

Q.2(v) Candidates who started with $\partial z/\partial x = \partial z/\partial y = 18$ were usually able to complete this successfully.

Q.3(i) Almost all candidates obtained the given values of t correctly. However, many candidates ignored the request to give the y-values.

Q.3(ii) Most candidates knew how to apply the parametric version of the formula to find the radius of curvature, and this was very often carried out accurately. The centre of curvature was sometimes omitted, and very often incorrect. A lot of candidates were unable to find the correct unit normal vector, and in particular there were very many sign errors.

Q.3(iii) Most candidates found the arc length correctly, although some were unable to simplify $\sqrt{((\mathrm{d}x/\mathrm{d}t)^2 + (\mathrm{d}y/\mathrm{d}t)^2)}$ to an form that can be integrated.

Q.3(iv) Most candidates selected the correct formula for the curved surface area, and the given answer was frequently obtained correctly.

Q.4(a)(i) Most candidates understood what was meant by associativity, although many demonstrated associativity of ordinary multiplication rather than multiplication modulo 16. Showing that P formed a group was done well, although those who did not exhibit the composition table often lost marks by not doing enough to demonstrate closure.

Q.4(a)(ii)-(iv) These parts were answered very well by most candidates.

Q.4(b) Most candidates gave a correct composition table; some gave a group with only the three elements e, a and b, and some gave a table which included more than four different elements. Most candidates explained that the group was non-cyclic because all the elements were self-inverse.

Q.4(c) Most candidates showed that **X**, **Y** and **Z** were self-inverse (and often completed the composition table) and deduced that G was isomorphic to the group in part (b). Most were able to specify an isomorphism, although some omitted to do so.

Q.4(d) Candidates were expected to explain why none of the four elements could be the identity element. Many did this well, although some lost marks for insufficient working, for example, simply stating that neither p nor q is the identity without further explanation. Several candidates based their argument on known group properties such as 'all groups of order four are abelian'. This was not given any credit as the question required reference to the group axioms.

Q.5(i) The transition matrix was almost always given correctly.

Q.5(ii) Most candidates used the fact that all the equilibrium probabilities were 1/3 to find the correct value $p = 0.25$.

Q.5(iii) This was very well understood, with just a few candidates using the fifth power of the transition matrix instead of the fourth.

Q.5(iv) Most candidates realised that the tasks chosen on day 7 and on day 4 were not independent, and therefore used the probabilities for day 4, together with the diagonal elements from the third power of the transition matrix, to obtain the required probability.

Q.5(v) The expected run length was very often found correctly.

Q.5(vi) Most candidates found the equilibrium probabilities successfully, some using the equilibrium equation, but most by evaluating high powers of the transition matrix. Some of those who obtained the limiting matrix did not then write down the equilibrium probabilities for tasks A, B and C.

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GCE Mathematics (MEI)

				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	57	52	47	42	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	56	49	42	35	29	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	58	52	47	42	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	64	57	51	45	39	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	59	53	48	43	38	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	60	54	48	43	38	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	60	54	49	44	39	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	67	61	55	49	43	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	58	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	59	53	47	41	36	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	60	53	46	40	34	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	55	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	59	52	46	40	34	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	60	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	60	54	48	42	37	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	48	43	38	34	30	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	55	50	45	40	36	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	55	49	44	39	33	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0

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		UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	59	52	46	40	34	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	59	52	46	40	34	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	48	43	38	34	30	0
		UMS	100	80	70	60	50	40	0

Level 3 Certificate and FSMQ raw mark grade boundaries June 2016 series

For more information about results and grade calculations, see www.ocr.org.uk/ocr-for/learners-and-parents/getting-your-results

Level 3 Certificate Mathematics for Engineering

			Max Mark	a*	a	b	c	d	e	u
H860	01 Mathematics for Engineering		This unit has no entries in June 2016							
H860	02 Mathematics for Engineering									

Level 3 Certificate Mathematical Techniques and Applications for Engineers

			Max Mark	a*	a	b	c	d	e	u
H865	01 Component 1	Raw	60	48	42	36	30	24	18	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI) (GQ Reform)

			Max Mark	a	b	c	d	e	u
H866	01 Introduction to quantitative reasoning	Raw	72	55	47	39	31	23	0
H866	02 Critical maths	Raw	60	47	41	35	29	23	0
		Overall	132	111	96	81	66	51	0

Level 3 Certificate Mathematics - Quantitive Problem Solving (MEI) (GQ Reform)

			Max Mark	a	b	c	d	e	u
H867	01 Introduction to quantitative reasoning	Raw	72	55	47	39	31	23	0
H867	02 Statistical problem solving	Raw	60	40	34	28	23	18	0
		Overall	132	103	88	73	59	45	0

Advanced Free Standing Mathematics Qualification (FSMQ)

			Max Mark	a	b	c	d	e	u
6993	01 Additional Mathematics	Raw	100	59	51	44	37	30	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

			Max Mark	a	b	c	d	e	u
6989	01 Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0

Version	Details of change
1.1	Correction to Overall grade boundaries for H866
	Correction to Overall grade boundaries for H867